

5.8/5.9 Practice #2

Advanced Pre-Calculus

1. 750 grams of a certain radioactive material decays to 500 grams in 3 days. Find how long it would take to become 300 grams.

$$\frac{500}{750} = \frac{750e^{k \cdot 3}}{750}$$

$$\frac{2}{3} = e^{3k}$$

$$\ln \frac{2}{3} = \frac{3k}{3} \quad k \approx -0.1352...$$

$$\frac{300}{750} = \frac{750e^{-0.1352t}}{750}$$

$$\frac{2}{5} = e^{-0.1352t}$$

$$\frac{\ln \frac{2}{5}}{k} = \frac{-0.1352t}{k} \quad t \approx 6.78 \text{ days}$$

2. A culture of 35 bacteria increases to 45 bacteria in 3 hours. How long would it take to become 110 bacteria?

$$\frac{45}{35} = \frac{35e^{k \cdot 3}}{35}$$

$$\frac{9}{7} = e^{3k}$$

$$\ln \frac{9}{7} = \frac{3k}{3} \quad k = 0.0838...$$

$$\frac{110}{35} = \frac{35e^{0.0838t}}{35}$$

$$\frac{110}{35} = e^{0.0838t}$$

$$\frac{\ln \frac{110}{35}}{k} = \frac{0.0838t}{k} \quad t \approx 13.67 \text{ hours}$$

3. A lasagna is removed from an oven when its temperature is 450°F in a room with a constant temperature of 70 °F. It takes 8 minutes for it to reach 400 °F. How long will it take for the lasagna to cool down to 175 °F?

$$400 = 70 + (450 - 70)e^{k \cdot 8}$$

$$330 = 380e^{8k}$$

$$\frac{33}{38} = e^{8k}$$

$$\ln \frac{33}{38} = \frac{8k}{8} \quad k = -0.0176...$$

$$175 = 70 + 380e^{-0.0176t}$$

$$105 = 380e^{-0.0176t}$$

$$\frac{105}{380} = e^{-0.0176t}$$

$$\frac{\ln \frac{105}{380}}{k} = \frac{-0.0176t}{k} \quad t \approx 72.94 \text{ minutes}$$

4. The following data represent crime rate against individuals (crimes per 1000 households) and their income (in dollars) in the United States in 2007.

- a. Use the graphing calculator to make a scatter plot of the data and sketch it here.



Income	Crime Rate
5000	213.1
11,250	201.3
20,000	167.0
30,000	154.6
52,500	151.2
62,500	144.6

- b. Using a graphing utility, build a logarithmic model from the data.

$$y = 459.334 - 28.751 \ln x$$

- c. Use your model to predict the crime rate of a household whose income is \$55,000.

$$y = 459.334 - 28.751 \ln 55000$$

$$y = 145.5 \text{ per } 1000 \text{ households}$$

5. The logistic mode $W(t) = \frac{14,656,248}{1 + 0.059e^{0.057t}}$ represents the number of farm workers in the United States t years after 1910.

a. How many workers were there initially? Round to the nearest person.

$$W(0) = \frac{14656248}{1 + 0.059e^{0.057(0)}} = \frac{14656248}{1 + 0.059} \approx 13,839,705 \text{ in } 1910$$

b. What is the limiting size for the number of farm workers in the United States?

$$14,656,248$$

c. How many farm workers were there in the United States in 2010?

$$t = 100 \quad W(100) = \frac{14656248}{1 + 0.059e^{0.057(100)}} \approx 786,567$$

d. When ~~did~~ the number of farm workers in the United States reach 10,000?

$$10000 = \frac{14656248}{1 + 0.059e^{0.057t}}$$

$$10000(1 + 0.059e^{0.057t}) = 14656248$$

$$10000 + 590e^{0.057t} = 14656248$$

$$\begin{array}{r} 14656248 \\ -10000 \\ \hline 14646248 \end{array}$$

$$\frac{590e^{0.057t}}{590} = \frac{14646248}{590}$$

$$\ln \frac{14646248}{590} = \frac{0.057t}{0.057}$$

$$t \approx 177 \text{ years}$$

$$+ 1910$$

$$\boxed{2087}$$